SEMINARIO

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Classical and Quantum Superintegrable Systems on Curved Spaces

Abstract: As models in Classical Mechanics, the two systems we will focus on, Darboux III and Taub-NUT, are maximally superintegrable Hamiltonian Systems, since they exhibit 2N -1 functionally independent integrals of motion. Moreover, they are defined on curved spaces with nonconstant scalar curvature, and possess hyperspherical symmetry. In fact they are two simple examples extracted from the Perlick’s class, consisting in turn of two different families, called Perlick I and Perlick II.

Indeed, the theoretical astrophysicist Volker Perlick published in 1992 a seminal paper on Classical and Quantum Gravity, answering to the following question: Is it possible to generalize the classical result obtained in 1873 by J.Bertrand, who proved that in a
3-dimensional flat space the only spherically symmetric maximally superintegrable hamiltonian systems were the Kepler-Coulomb and the (isotropic) Harmonic oscillator, entailing that they were the sole systems such that all their bounded trajectories were closed (and stable periodic orbits did exist)?

V.Perlick had to give the flatness condition up, still requiring however a conformally flat and spherically symmetric space, and found two 4-parameter classes of metrics and associated potentials were the maximal superintegrabilty was preserved. We have called them Bertrand Spaces. It is worth noticing that Perlicks results unveiled the deep connection between geometric and dynamical features, quite in the spirit of General Relativity.

We point out that the classification of Perlick’s systems in two different families has been done according to specific properties of the Hamiltonians. The systems belonging to Family I have been denoted as intrinsic Kepler systems, because the associated potentials are the fundamental solutions (the Green’s functions) in dimension 3, of the Laplace-Beltrami operators associated with the metric I, playing the same role as 1/r in a flat space. The systems belonging to Family II will be denoted as intrinsic oscillators because, up to multiplicative and additive constants, the corresponding potentials are the inverse squared of those pertaining to Family I. We will mostly investigate the Familiy II systems, namely Darboux III and Taub-NUT. Both those systems are one-parameter deformations of the corresponding flat ones. For a special value of the parameter, typically 0, one recovers the euclidean case.

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