SEMINARIO

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Numerical Investigation of Crouzeix's Conjecture

Abstract: Crouzeix's conjecture is among the most intriguing developments in matrix theory in recent years. Made in 2004 by Michel Crouzeix, it postulates that, for any polynomial p and any matrix A, $\|p(A)\| \leq 2\max(|p(z)| : z \in W(A))$, where the norm is the 2-norm and $W(A)$ is the field of values (numerical range) of A, that is the set of points attained by $v*Av$ for some vector v of unit length. Crouzeix proved in 2007 that the inequality above holds if 2 is replaced by 11.08, and very recently this was greatly improved by Palencia, replacing 2 by $1+\sqrt{(2)}$. Furthermore, it is known that the conjecture holds in a number of special cases, including $n=2$. We use nonsmooth optimization to investigate the conjecture numerically by attempting to minimize the "Crouzeix ratio", defined as the quotient with numerator the right-hand side and denominator the left-hand side of the conjectured inequality. We present numerical results that lead to some theorems and further conjectures, including variational analysis of the Crouzeix ratio at conjectured global minimizers. All the computations strongly support the truth of Crouzeix's conjecture.

This is joint work with Anne Greenbaum and Adrian Lewis.

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