

## ATENEO



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## ***On distribution and quantile functions in $\mathbb{R}^d$ : a measure-transportation approach***

**Abstract:** Unlike the real line, the real space  $\mathbb{R}^d$ ,  $d \geq 2$  is not 'naturally' ordered. As a consequence, such fundamental univariate concepts as quantile and distribution functions, ranks, signs, all order-related, do not straightforwardly extend to the multivariate context. Since no universal pre-existing order exists, each distribution, each data set, has to generate its own: the rankings behind any sensible concept of distribution or quantile function, ranks, or signs, inherently will be distribution-specific and, in empirical situations, data-driven.

Many proposals have been made in the literature for such orderings---all extending some aspects of the univariate concepts, but failing to preserve the essential properties that make classical rank-based inference a major inferential tool in the analysis of semiparametric models where the density of some underlying noise remains unspecified: (i) exact distribution-freeness, and (ii) asymptotic semiparametric efficiency, see Hallin and Werker~(Bernoulli 2003).

Quantile functions and their empirical counterparts (ranks, signs, and empirical quantiles) are well understood and well developed for univariate observations. We use the close connection between those classical concepts and measure transportation results, to define fully general concepts of distribution and quantile functions (along with their empirical counterparts---called the *Monge-Kantorovich ranks and signs*) coinciding, in the univariate setting, with the traditional concepts, and enjoying under completely unspecified (absolutely continuous)  $d$ -dimensional distributions, the essential properties ((i) and (ii)) that make traditional rank-based inference an essential tool in semiparametric inference.

Based on Chernozhukov, V., A. Galichon, M. Hallin, and M. Henry (2017) Monge-Kantorovich depth, quantiles, ranks, and signs, Ann. Statist. 45, 223-256.

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