SEMINARIO

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On the non-triviality of Gelfand-Shilov spaces

Abstract: Let $(M_p)_{p\in\mathbb{N}}$ and $(A_p)_{p\in\mathbb{N}}$ be two sequences of positive real numbers. The space $\mathcal{S}^{\{M_p\}}_{\{A_p\}}$ consists of all $\varphi \in C^\infty(\mathbb{R})$ such that

$$\sup_{p,q\in\mathbb{N}} \sup_{x\in\mathbb{R}^d} \frac{|x^q \varphi^{(p)}(x)|}{h^{p+q} M_p A_q} < \infty \qquad (*)$$

for some $h > 0$. Similarly, the space $\mathcal{S}^{(M_p)}_{(A_p)}$ consists of all $\varphi \in C^\infty(\mathbb{R})$ such that $(*)$ holds for all $h > 0$. These spaces were introduced by Gelfand and Shilov in the 1950's and, in their honour, they are nowadays called Gelfand-Shilov spaces. Gelfand and Shilov put forward the problem of characterizing the non-triviality of the spaces $\mathcal{S}^{\{M_p\}}_{\{A_p\}}$ and $\mathcal{S}^{(M_p)}_{(A_p)}$ in terms of the sequences $(M_p)_{p\in\mathbb{N}}$ and $(A_p)_{p\in\mathbb{N}}$. They themselves showed that $\mathcal{S}^{\{p!^\alpha\}}_{\{p!^\beta\}}$, $\alpha, \beta > 0$, is non-trivial if and only if $\alpha + \beta \geq 1$ but, in general, the problem still seems to be open. In the first part of this talk I will give an overview of several known partial solutions to this problem while, in the second part, I will report on recent work (joint with Jasson Vindas) in which we completely solved the problem for $M_p = p!$ fixed.

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