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## ATENEEO



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### ***Around the Banach-Tarski paradox***

**Abstract:** There is a classical mathematical theorem (due to Banach and Tarski) that implies the following shocking statement:

*An orange can be divided into finitely many pieces, these pieces can be rotated and rearranged in such a way to yield two oranges of the same size as the original one.*

This apparent paradoxical decomposition of a three dimensional ball will be the starting point of the talk.

In 1929 J. von Neumann recognizes that one of the reasons underlying the Banach-Tarski paradox is the fact that on the unit ball there is an action of a discrete subgroup of isometries that fails to have the property of *amenability* (*promediabilidad* in Spanish or *Mittelbarkeit* in German). Since then, the notion of amenability has become ubiquitous in mathematics. Amenability is a central concept in group theory which also has ramifications measure theory, geometry or operator theory.

In the second part of the talk we will address more recent developments in relation to the dichotomy amenability vs. paradoxical decompositions in different mathematical situations like, e.g., for metric spaces or operator algebras. Time permitting we will present a result unifying all these approaches in terms of the so-called Roe algebras.

**Sala de Grados II, Facultad de Ciencias**  
**Jueves 23 de Enero de 2020 (17:00)**

