SEMINARIO

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Old and new spectral problems for the prolate spheroidal equation. An unified analytical approach. Application to the Connes-Moscovici spectrum which matches the zeroes of zeta

Abstract: A joint work with Françoise Richard-Jung and Jean Thomann. The main actor in the play is the prolate spheroidal operator (of zero order) $W_\Lambda = -\frac{d}{dx}(\Lambda^2 - x^2)\frac{d}{dx} + (2\pi\Lambda x)^2$; Λ is a parameter (bandwith parameter). It is real in a first step and after complex; W_Λ is formally self-adjoint. The spheroidal operators appeared in the study of the Helmoltz equation $(\Lambda + k^2)\Psi = 0$ on a spheroid (prolate resp. oblate). The prolate spheroid is a rugby ball. There is a separation of variables in prolate (resp. oblate) coordinates. The solutions of angular and radial equations are the spheroidal functions. They generalize the spherical functions (football ball).

I will recall some results in signal theory due to Slepian and all in Bell Labs 1960-1965. Signals cannot be perfectly localized in time and frequency and there is a problem in communication of signals : limited time, limited range of frequencies. This lead to the study of an integral convolution operator Q_Λ with a sine cardinal kernel. Its first eigenvalues are very near of 1 and afterwards they fall abruptly and became very small. Then the study of eigenfunctions become difficult. Slepian and all made a remarkable discovery : the operator Q_Λ commutes with the prolate differential operator W_Λ. Then the eigenfunctions are the prolate spheroidal functions and it is possible to use a singular version of the Sturm-Liouville theory on $[-\Lambda, \Lambda]$ for their study. Slepian called this "the lucky accident" and said that it remains a mystery. I will try to explain the origin of the miracle.

I will present a new theory of spectra of rational linear second order operators elaborated with Françoise and Jean. We choose singular points as boundaries and we define boundary conditions. in the regular and irregular cases.

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