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The volume polynomial and the Heine-Shephard problem

Abstract: It is well-known that if you rescale a d-dimensional body K by a factor of t, its volume gets multiplied by t^d. In other words, the volume of tK is a homogeneous polynomial of degree d in t. In 1903, Minkowski gave a natural multivariate generalization of this fact: Given convex bodies $K_1,\dots,K_n$ in $\mathbb{R}^d$, the volume of $t_1K_1+\cdots+t_nK_n$ is a degree d homogeneous polynomial in the scalars $t_1,\dots,t_n$. Can one describe, for fixed values of n and d, the space of all volume polynomials in terms of algebraic inequalities for the coefficients? Surprisingly, except when $n=3$ and $d=2$ (Heine, 1938), and $n=2$ and any d (Shephard, 1960), the question remains wide open. The study of inequalities for the coefficients of the volume polynomial (mixed volumes) is at the core of the Brunn-Minkowski theory of convex bodies. The most famous one is the Aleksandrov-Fenchel inequality -- a far-reaching generalization of the isoperimetric inequality. Shephard derived many new inequalities from the Aleksandrov-Fenchel inequality, but they do not describe the space of volume polynomials beyond the known cases. I will present a way of deriving new inequalities for mixed volumes using polyhedral geometry and combinatorics of the real Grassmannian. This approach, in particular, allows us to solve a weaker version of the Heine-Shephard problem in the case of $n=4$ and $d=2$. This is joint work with Gennadiy Averkov, Katherina von Dichter, and Simon Richard.

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