SEMINARIO

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What do Betti diagrams of edge ideals look like?

Abstract: Given an arbitrary field k and a graph G with vertex set $\{x_1,\dots,x_n\}$ and edge set $E(G)$, it is possible to define the edge ideal $I(G)$ associated to G in the polynomial ring $R=k[x_1,\dots,x_n]$ as

$$I(G)=\langle x_ix_j \mid \{x_i,x_j\}\in E(G)\rangle\,.$$

These ideals were first defined in 1990 by R. H. Villarreal, establishing a strong connection between commutative algebra and graph theory. Since then, several authors began to study them by taking advantage of the properties of G and other graphs associated to $I(G)$.

In 1990, R. Fröberg proved the first very relevant result in this direction: $I(G)$ has a linear resolution, meaning that its Betti diagram has only one row, if and only if the complement G^c of G has no induced cycles of length at least 4. This result was later improved by D. Eisenbud, M. Green, K. Hulek and S. Popescu, who determined the step at which the resolution of $I(G)$ stops being linear in combinatorial terms. In 2009 and 2014, Ó. Fernández-Ramos and P. Gimenez were able to generalize these results focusing on edge ideals associated to bipartite graphs.

In this talk, we will give a review of the state of the art in the study of Betti diagrams of edge ideals associated to graphs, and we will pay special attention to the combination of homological and combinatorial tools that has played an important role in this story. In addition, we will discuss new results related to the application of homological techniques to edge ideals of families of graphs with certain characteristics.

This talk is based on joint work with Ignacio García-Marco and Philippe Gimenez.

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